

Unit II: Free-fall and Projectile Motion

Aristotle

From the time of Aristotle (384-322 BC) until the late 1500's, gravity was believed to act differently on different objects.

- This was based on Aristotle's observations of doing things like dropping a metal bar and a feather at the same time. Which one hits the ground first?
 - Obviously, common sense will tell you that the bar will hit first, while the feather slowly flutters to the ground.
- In Aristotle's opinion, this was because the bar was being pulled harder (and faster) by gravity because of its physical property of having more mass.
- Because everyone could see this when they dropped different objects, it wasn't questioned for almost 2000 years.

Galileo

Galileo Galilei was the first major scientist to refute (prove wrong) Aristotle's theories.

- In his famous (at least to Physicists!) experiment, Galileo *supposedly* went to the top of the leaning tower of Pisa and dropped a wooden ball and a lead ball at the same time.
 - Both were the same size, but different masses.
- Down below an assistant watched for them to hit the ground.
 - They both hit the ground at the same time, even though Aristotle would say that the heavier metal ball should hit first.

Galileo had shown that the different rates at which some objects fall is due to air resistance, a type of friction.

- Get rid of friction (air resistance) and all objects will fall at the same rate.
- Galileo said that the acceleration of any object (in the absence of air resistance) is the same.

To this day we follow the model that Galileo created.


In the absence of air resistance, all objects fall at a constant acceleration, regardless of their mass.

This acceleration is called the acceleration due to gravity.

$$\text{Acceleration Due to Gravity} = |\vec{a}_g| = g = 9.80 \text{ m/s}^2$$

In any of the questions we will do, we must assume that the entire problem happens near the surface of the Earth. We will learn in a later unit that the acceleration due to gravity does decrease as you move further away from the centre of the earth. The magnitude of g also changes on different celestial bodies.

Since gravity is just an acceleration like any other, it can be used in any of the formulas that we have used so far. Just be careful about using the correct sign (positive or negative) for the variables in the problem. I strongly suggest that you stick with **up** being **positive** and **down** being **negative**.

 Upwards + velocity + displacement	 Downwards - velocity - displacement - gravity
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Ex. A ball is thrown up into the air with an initial velocity of 26.3 m/s. Determine its velocity after 1.80 s have passed.

$$\begin{aligned}
 \vec{v}_1 &= 26.3 \text{ m/s} & \vec{v}_2 &= \vec{v}_1 + \vec{a} \Delta t \\
 \vec{v}_2 &=? & \vec{v}_2 &= 26.3 \text{ m/s} + (-9.80 \text{ m/s}^2)(1.80 \text{ s}) \\
 \Delta t &= 1.80 \text{ s} & \vec{v}_2 &\approx 8.66 \text{ m/s} \\
 \vec{a} &= -9.80 \text{ m/s}^2
 \end{aligned}$$

Ex. I throw a ball down from the top of a cliff so that it leaves my hand moving at 12 m/s. Determine how fast it is going 3.47 s later.

$$\begin{aligned}
 \vec{v}_1 &= -12 \text{ m/s} & \vec{v}_2 &= \vec{v}_1 + \vec{a} \Delta t \\
 \vec{v}_2 &=? & \vec{v}_2 &= -12 \text{ m/s} + (-9.80 \text{ m/s}^2)(3.47 \text{ s}) \\
 \Delta t &= 3.47 \text{ s} & \vec{v}_2 &\approx -46 \text{ m/s} \\
 \vec{a} &= -9.80 \text{ m/s}^2
 \end{aligned}$$

Ex. A jackalope jumps up into the air. As he leaves the ground he is travelling at 16.0 m/s. How fast is he going after 2.8 s?

$$\begin{aligned}
 \vec{v}_1 &= 16.0 \text{ m/s} & \vec{v}_2 &= \vec{v}_1 + \vec{a} \Delta t \\
 \vec{v}_2 &=? & \vec{v}_2 &= 16.0 \text{ m/s} + (-9.8 \text{ m/s}^2)(2.8 \text{ s}) \\
 \Delta t &= 2.8 \text{ s} & \vec{v}_2 &\approx -11 \text{ m/s} \\
 \vec{a} &= -9.80 \text{ m/s}^2
 \end{aligned}$$

Throwing an Object Up Into the Air

There are a few rules that you have to keep track of. Let's look at the way an object thrown up into the air moves.

Going up...

- 1) It starts at the bottom at the maximum speed.
- 2) As it rises, it slows down because gravity is a negative acceleration.
- 3) It reaches its maximum height, where for a moment its instantaneous velocity is zero. This is exactly half ways through the flight time if it is landing at the same height it was thrown.

Coming down...

- 1) The ball begins to speed up, but downwards. Because gravity is negative, the velocity of the object increases in the negative direction.
- 2) When it reaches the same height that it started from (like the ground, or the person's hand), it will be going at the same speed down as it was originally moving up at. The only difference is that this velocity is negative because it is pointing down.
- 3) It takes just as much time to come down as it did to go up if it is landing at the same height it was thrown.

Applying these rules might seem complicated, but since they stay the same all the time you can get used to the problems by just practicing them over and over again.

Ex. I throw my ball up (again) at a velocity of 12 m/s.

- a) Determine how much time it takes to reach its maximum height.

$$\begin{aligned} \vec{v}_1 &= 12 \text{ m/s} & \vec{a} &= \frac{\Delta \vec{v}}{\Delta t} & \Delta t &= \frac{0 \text{ m/s} - 12 \text{ m/s}}{-9.80 \text{ m/s}^2} \\ \Delta t &= ? & & & & \\ \vec{a} &= -9.80 \text{ m/s}^2 & \Delta t &= \frac{\Delta \vec{v}}{\vec{a}} & \Delta t &\approx 1.2 \text{ s} \\ \vec{v}_2 &= 0 \text{ m/s} & & & & \end{aligned}$$

- b) Find how high the ball goes.

$$\begin{aligned} \Delta d &= ? & \vec{v}_2^2 &= \vec{v}_1^2 + 2\vec{a}\Delta d & - \frac{(12 \text{ m/s})^2}{2(-9.8 \text{ m/s}^2)} &= \Delta d \\ & & \frac{\vec{v}_1^2}{2\vec{a}} &= \Delta d & \Delta d &= 7.3 \text{ m} \end{aligned}$$

c) Determine how fast it is going when it reaches my hand again.

Shortcut: same as initial speed, but opposite direction

$$V_2 = -12 \text{ m/s} \\ \text{or } 12 \text{ m/s [down]}$$

Gee's

You might have heard people flying fighter jets or rockets in movies say how many "gee's" they were feeling. All this means is that they are comparing the acceleration they are feeling to regular gravity.

So, right now just sitting in a chair, you are experiencing 1 gee - regular gravity. This means that you are experiencing one times the acceleration of gravity. One times 9.80 m/s^2 is equal to 9.80 m/s^2 .

During lift-off the astronauts in the space shuttle experience about 4 gee's. That works out to about $4 \times 9.80 = 39 \text{ m/s}^2$.

Ex. Acceleration due to gravity on the moon is 1.67 m/s^2 . Determine how many gee's this is.

$$\frac{1.67 \text{ m/s}^2}{9.80 \text{ m/s}^2} = 0.17 \text{ gee's}$$

Ex. A space probe sent to one of Jupiter's moons, Callisto, is taking pictures with a digital camera that records one picture every tenth of a second (measured to a precision of three sig digs). While doing this, a small screw falls off a part of the probe right in front of the camera and can be seen to start falling past the camera as it takes the pictures. Over a series of six pictures, the screw can be seen to fall 15.7 cm starting from rest. Determine how many gee's there are on this moon.

Projectiles Launched Horizontally

A projectile is any object which is thrown or otherwise projected into the air. Again, we assume that the effects of air resistance are negligible for the problems we deal with.

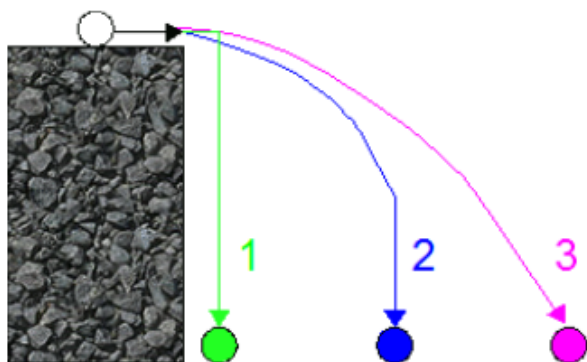
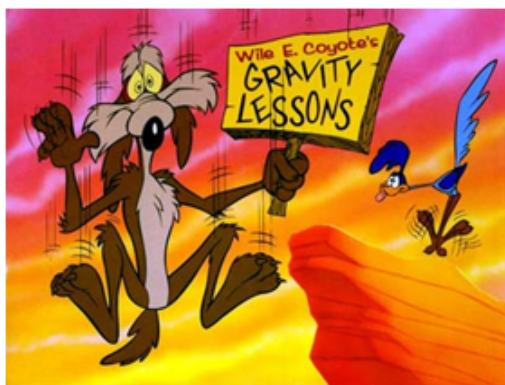
The study of projectile motion brings together a lot of what you have learned in the past few sections. You need to know about gravity, velocity, acceleration, and vector components to be able to fully understand (and figure out) these questions. Don't worry, though. Even with all that stuff to keep track of, learning how to do these questions and understand what is happening a little bit at a time makes it all manageable.

Wile E. Coyote

I'm sure you have seen the cartoon where the Coyote is chasing after the Road Runner and runs off of the cliff. He hangs in midair for a second, looks down, and then starts to fall. The question is how true is this, and how many people believe it is true?

A few years ago some researchers in the U.S. went to elementary schools, junior and senior high schools, and universities and asked them to look at the following:

"Ignoring air resistance, which of the following correctly shows what an object would do if it rolled off a cliff?"



The correct answer is actually number 3, and if you think about it using the physics you've studied it makes sense.

Let's say a coyote does run off a cliff. As he leaves the cliff he has a horizontal velocity. From studying forces and acceleration you already know that the only way to change that horizontal velocity (cause a horizontal acceleration) is to exert a horizontal force on the coyote.

If we are ignoring air resistance (which is a very good idea since it will be practically zero), then there is no horizontal force to cause a horizontal acceleration. Since there is no horizontal acceleration, the coyote will travel horizontally at the same speed the whole time!

That doesn't tell us anything about what is happening vertically, which is completely separate from what the object is doing horizontally.

As soon as the coyote leaves the cliff he will experience a vertical force due to gravity. This force will cause him to start to accelerate in the vertical direction. As he falls he will be going faster and faster in the vertical direction.

Looking at this problem as what is happening horizontally and vertically, you should get the idea that this is just like the components of vectors that we were just working on a couple of lessons back.

The horizontal and vertical components of the motion of an object going off a cliff are separate from each other, and cannot affect each other. In a lot of books you will see the horizontal component called x and the vertical component called y .

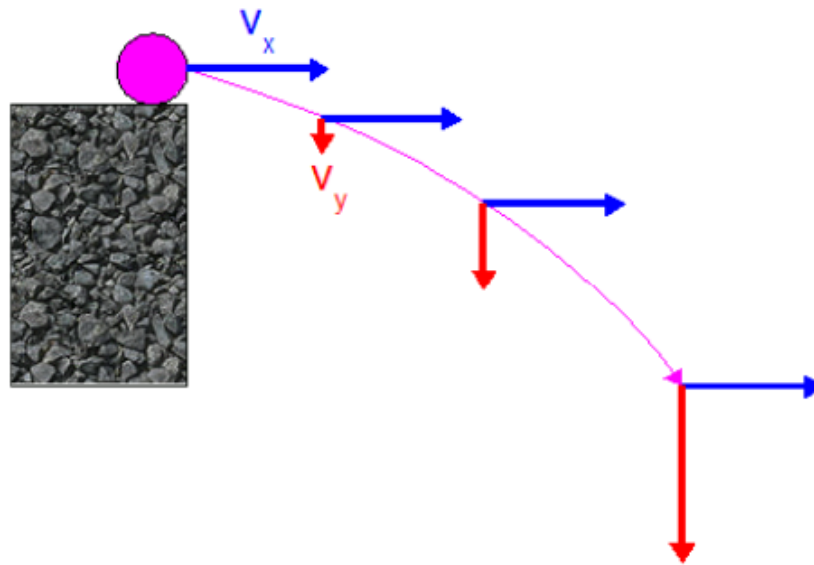


Illustration 2: Components as a ball rolls off a cliff.

The x -component is there from the start, and stays the same the entire time.

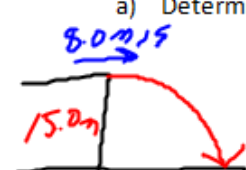
The y -component doesn't even exist at the beginning, but grows bigger as the object falls.

The shape of the path that it follows is actually a parabola. If you've studied those in math class, great! Don't worry about it if you haven't, just as long as you recognize the shape and know the name.

To understand how to actually figure out questions involving these situations, it's probably best to look at an example. Keep in mind the characteristics of the object as it falls while you go through the example. When you are doing a part of a question that has to do with vertical movement "THINK VERTICAL" and only use vertical ideas (like gravity). When you are doing a part of a question that has to do with horizontal movement "THINK HORIZONTAL" and only use horizontal ideas (no gravity/acceleration).

Ex. I throw a ball off the edge of a 15.0m tall cliff. I threw it horizontally at 8.0 m/s.

a) Determine how much time it takes to fall.



Think vertically!

$$\Delta y = -15.0\text{m}$$

$$\vec{v}_x = 8.0\text{m/s}$$

$$\vec{a}_y = -9.81\text{m/s}^2$$

$$\vec{v}_{y0} = 0\text{m/s}$$

$$\Delta y = \vec{v}_{y0}\Delta t + \frac{1}{2}\vec{a}_y(\Delta t)^2$$

$$\Delta y = \frac{1}{2}\vec{a}_y(\Delta t)^2$$

$$\sqrt{\frac{2\Delta y}{\vec{a}_y}} = \Delta t$$

$$\Delta t = \sqrt{\frac{2(-15.0\text{m})}{-9.81\text{m/s}^2}}$$

$$\Delta t = 1.7487\text{s}$$

$$\Delta t \approx 1.75\text{s}$$

b) Determine how far from the base of the cliff it hits the ground.

Think horizontal!

$$\Delta x = ?$$

$$\vec{v}_x = 8.0\text{m/s}$$

$$\Delta x = \vec{v}_x(\Delta t)$$

$$\Delta x = 8.0\text{m/s}(1.7487\text{s})$$

$$\Delta x = 13.9896\text{m}$$

$$\Delta x \approx 14.0\text{m}$$

c) Determine how fast it is moving vertically when it hits the ground.

$$\vec{v}_{y2} = ?$$

$$\vec{v}_{y1} = 0\text{m/s}$$

$$\vec{a}_y = -9.81\text{m/s}^2$$

$$\Delta t = 1.7487\text{s}$$

$$\Delta y = -15.0\text{m}$$

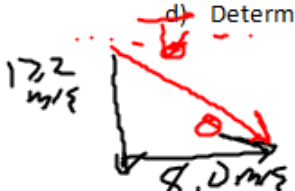
$$\vec{v}_{y2}^2 = \vec{v}_{y1}^2 + 2\vec{a}_y\Delta y$$

$$v_{y2} = \sqrt{2(-9.81\text{m/s}^2)(-15.0\text{m})}$$

$$v_{y2} = -17.1552\text{m/s}$$

$$v_{y2} \approx -17.2\text{m/s}$$

d) Determine what its total velocity is when it hits the ground.



$$v = \sqrt{(17.2)^2 + (8.0)^2}$$

$$v \approx 19\text{m/s}$$

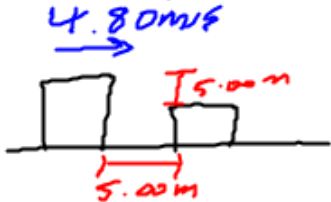
$$\theta = \arctan\left(\frac{17.2}{8}\right)$$

$$\theta = 66^\circ$$

$$\vec{v} = 19\text{m/s} [66^\circ \text{ down from the horizontal}]$$

Although there will always be slight differences in actual problems, this is the standard sort of question that you will be asked for these types of problems.

Ex. A stuntman is to run across a rooftop and then jump over a 5.00 m gap landing on the roof of the next building. The building he is jumping from is 5.0 m higher than the next building. If the stuntman is running at 4.80 m/s will he make it?



4.80 m/s
 5.00 m
 5.00 m
 $u_x = 4.80 \text{ m/s}$
 $\Delta d_y = -5.00 \text{ m}$
 $v_{xy} = 0 \text{ m/s}$
 $a_y = -9.81 \text{ m/s}^2$
 $\Delta d_x = ?$

Time to fall:

$$\Delta d_y = v_{xy} \Delta t + \frac{1}{2} a_y (\Delta t)^2$$

$$\Delta d_y = \frac{1}{2} a_y (\Delta t)^2$$

$$\sqrt{\frac{2 \Delta d_y}{a_y}} = \Delta t$$

$$\Delta t = \sqrt{\frac{2(-5.00 \text{ m})}{-9.81 \text{ m/s}^2}}$$

$$\Delta t = 1.0096 \text{ s}$$

Range:

$$\Delta d_x = v_x \Delta t$$

$$\Delta d_x = 4.80 \text{ m/s} (1.0096 \text{ s})$$

$$\Delta d_x = 4.84608 \text{ m}$$

$$\Delta d_x \approx 4.85 \text{ m}$$

$\Delta d_x < \text{Gap}$
 $4.85 \text{ m} < 5.00 \text{ m}$

... will not make it.

Summary:

- Horizontally, a projectile travels at a constant velocity.
- Vertically, a projectile travels at the same rate as an object experiencing free-fall motion.
- To analyze projectile motion it is useful to consider the vertical (y) and horizontal (x) components of the velocity and displacement separately.
- The horizontal motion of a projectile depends on the horizontal component of the initial velocity (v_x).
- The horizontal displacement (Δd_x) of a projectile, relative to the ground, can be found using the horizontal component of the velocity and the total flight time (Δt). This displacement is called range.
- The vertical motion of a projectile depends on the vertical component of the initial velocity and on the acceleration due to gravity.
- The vertical motion of a projectile can be determined with the equations for free-fall motion using vertical components and the acceleration due to gravity.

Projectile Motion at an Angle

To do questions involving objects launched from the ground upwards at an angle (like kicking a football up into the air and watching it as it arcs in the air and comes back down), you need to add a few more steps to the way you did the questions for objects launched horizontally.

There are actually two ways to do these types of problems, one based on the vertical velocity of the object, the other based on the vertical displacement.

The only big difference in these methods is how we are going to calculate the time that the object spends in the air. Choose whichever method you are most comfortable with, and whichever one suits the particular question you are doing. We will look at each and then it's up to you to figure out which way you will approach a problem.

Imagine for a moment that you are watching an object as it rises into the air after you kick it upwards at an angle. Look at Illustration 1 below as you read through this description.

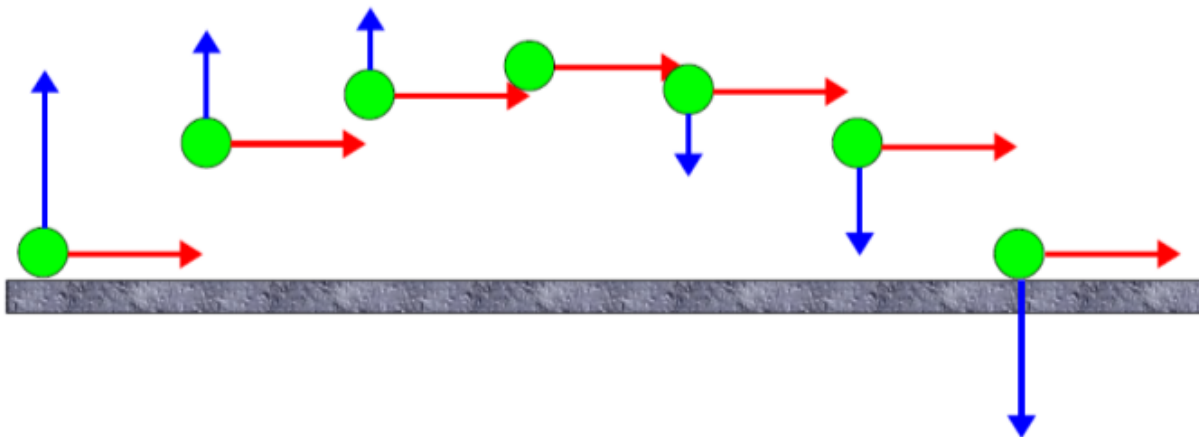


Illustration 1: Object launched at an angle.

When it left your foot, it was going at the fastest that it can possibly move during its flight. The instant it leaves your foot, gravity is pulling down on it, causing it to have less and less vertical velocity. Remember that there will be no change in the horizontal component of its velocity.

When it reaches the highest point in its flight, it isn't moving up, and it isn't moving down, for an instant of time. Its vertical velocity is zero!

By the time it reaches the ground again, it will still be moving with its original horizontal velocity and will have just as much vertical velocity as when it left your foot. It will have the exact same velocity as it left your foot with.

We can use this information about its vertical movement to do some calculations. We know:

- that there is gravity (-9.80 m/s^2) causing the acceleration on the object vertically.
- the initial vertical velocity of the object.
- the final vertical velocity of the object. We can even use this two ways, since we can say that the final vertical velocity happens at the halfway point (zero m/s), or when it gets back to the ground (same as it left the ground at).



This gives us enough information to calculate the maximum height of the flight, and the time it spends in the air. After that, we can calculate just about anything.

Ex. You kick a soccer ball at an angle of 40° above the ground with a velocity of 20 m/s . Determine:

a) How high will it go?

Handwritten notes:

Think Vertical

20 m/s

40°

V_x

V_y

$V_y = 20 \text{ m/s}$

$a_y = -9.81 \text{ m/s}^2$

$V_{y2} = 0$

$\Delta d_y = ?$

$V_y = 20 \text{ m/s} \sin(40^\circ)$

$V_y = 12.8558 \text{ m/s}$

$V_x = 20 \text{ m/s} \cos(40^\circ)$

$V_x = 15.3204 \text{ m/s}$

Handwritten equations:

$V_{y2}^2 = V_{y1}^2 + 2a_y \Delta d_y$

$0 = V_y^2 + 2a_y \Delta d_y$

$-\frac{V_y^2}{2a_y} = \Delta d_y$

$\Delta d_y = -\frac{(12.8558 \text{ m/s})^2}{2(-9.81 \text{ m/s}^2)}$

$\Delta d_y \approx 8.42363 \text{ m}$

$\Delta d_y \approx 8 \text{ m}$

think vertical

b) How much time does it spend in the air?

Find Top

$$v_{y1} = 12.8558 \text{ m/s}$$

$$v_{y2} = 0 \text{ m/s}$$

$$a_y = -9.81 \text{ m/s}^2$$

$$\Delta t = ?$$

$$\vec{a}_y = \frac{v_{y2} - v_{y1}}{\Delta t}$$

$$\Delta t = \frac{-v_{y1}}{\vec{a}_y}$$

$$\Delta t = 2(1.31048 \text{ s})$$

$$\Delta t = 2.62097 \text{ s}$$

$$\Delta t \approx 3 \text{ s}$$

$$\Delta t = -\frac{12.8558 \text{ m/s}}{-9.81 \text{ m/s}^2}$$

$$\Delta t = 1.31048 \text{ s} \rightarrow \text{up + time}$$

c) How far away from you will it hit the ground (aka range)?

think horizontal

$$v_x = 15.3209 \text{ m/s}$$

$$\Delta t = 2.62097 \text{ s}$$

$$\Delta x = ?$$

$$\vec{\Delta x} = \vec{v}_x \Delta t$$

$$\Delta x = 15.3209 \text{ m/s} (2.62097 \text{ s})$$

$$\Delta x \approx 40.15561 \text{ m} \approx \boxed{40 \text{ m}}$$

d) What is the ball's velocity when it hits the ground?

$$20 \text{ m/s} [40^\circ \text{ below horizontal}]$$

Ex. A basketball player tries to make a half-court jump shot, releasing the ball at the same height as the basket. Assuming the ball is launched 14 m from the basket at 51° , find the velocity the player must shoot the ball.